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Converting Covid-19 Deterministic Model to its Stochastic Counterpart

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ABSTRACT

Deterministic models have played an important role in biology over the past few centuries, and continue to play. However, there is a need to introduce a random factor due to the nature of biological processes, which led to the emergence of stochastic models. To bridge the gap between deterministic and stochastic models, and in appreciation of the huge efforts made in deterministic models, which have played a fundamental role in understanding, predicting, and controlling the transmission dynamics of infectious diseases, this study aims to clarify a method for creating stochastic model for one of most vital and recent deterministic, that is Covid-19 model with two doses, which showed the effect of receiving both doses in containing the disease. To achieve this goal, the deterministic model was studied extensively before converting to corresponding stochastic. The deterministic and stochastic systems were solved numerically and represented using MATAB tools.

Keywords: Covid-19 deterministic model, stochastic model, next generation matrix method (NGMM), The diffusion matrix G.

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تحويل نموذج كوفيد–19 الحتمي (المحدد) الى نظيره العشوائي

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الملخص

لعبت النماذج المحددة دورًا مهمًا في الرياضيات الحيوية على مدى القرون الخمسة السابقة، ومازالت تلعب هذا الدور وتظهر نتائج طيبة، إلا أنه هناك حاجة لإدخال العامل العشوائي نظرا لطبيعة العمليات البيولوجية، مما أدى إلى ظهور النماذج العشوائية.

ولجسر الهوة بين النماذج المحددة والعشوائية، وتقديراً للجهود الضخمة التي بذلت في صياغة النماذج المحددة، فإن هذه الدراسة تهدف لإيضاح طريقة لإيجاد نموذج عشوائي مناظر لأحد أحدث النماذج المحددة وأكثرها حيوية، ألا وهو كوفيد–19 بجرعتين من التطعيم. ولإحراز هذا الهدف فإن تمت دراسة النموذج المحدد باستفاضة قبل تحويله إلى نظيره العشوائي. كل ذلك تم بيانه بطريقة مستحدثة وكلا النظامين تم حلهما عدديا وإبراز الحلول باستخدام أدوات برنامج الماتلاب.

الكلمات المفتاحية: نموذج كوفيد–19 المحدد، النموذج العشوائي، طريقة مصفوفة الجيل التالي، مصفوفة الانتشار.

1-Introduction:

More than 20 infectious agents have been identified as a result of numerous disease outbreaks that have occurred worldwide over the past 10 years, Communities are in chaos due to these infectious diseases. Emerging infections in specific areas have the capacity to spread quickly across national boundaries and pose a serious threat to public health, Covid-19 is a clear example of this.[1]

SARS-CoV-2, a novel coronavirus that arose in Wuhan, China, at the end of 2019, is the source of COVID-19 infections. Following its worldwide spread, it resulted in an unprecedented pandemic never before seen since the 1918 influenza pandemic (3–8). Sars-Cov-2 is a highly transmissible virus that mostly spreads by droplets

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(reproduction number R_0: 2-3). This means that one infected individual has the potential to infect up to three more individuals. [1-4]. People who have minor symptoms or no symptoms at all may be the ones spreading the disease. Coronavirus infection is a respiratory disease that infects multiple organs and tissues and sets off a chain of events that affects the entire body [1,2]. On January 30, the World Health Organization deemed COVID-19 a public health emergency of global significance, and on March 11, it was deemed a pandemic. The terrible worldwide effects of the ongoing COVID-19 pandemic serve as a warning about the potential dangers of newly developing infectious illnesses. On April 04, 2022, there have been 492,271,251 confirmed cases worldwide with 6,178,291 deaths and 427,442,919 recovered. These numbers are exponentially growing day by day. On March, 07, 2023, there have been 680,817,071 confirmed cases worldwide with 6,806,074 deaths and 653,716,966 recovered [3]. However, after strenuous efforts of precautionary and medical processes that culminated in discovery of vaccinations, the intensity of the epidemic began to wane. In the last update on April 13, 2024, there have been 704,753,890 confirmed cases worldwide with 7,010,681 deaths and 675,619,811 recovered, according to the (worldometers.info) website: (https://www.worldometers.info/coronavirus/).
It took the scientific community a few weeks to define the epidemic and identify the causal agent, as well as the creation of extremely specialized diagnostic techniques. As for coronavirus (COVID-19), there are no established therapies. Still, an enormous number of investigation systems are being investigated.[1]. The World Health Organization has started a global clinical study known as the solidarity study in an attempt to aid in the search for a COVID-19 cure. A Covid-19 vaccination can prevent Covid-19 sickness, and the hunt for a SARS-CoV-2 vaccine is a top priority. Getting vaccinated can lessen the severity of disease, also advancement and widespread accessibility of vaccines are necessary for the COVID-19 epidemic to come to an end. For the pandemic's several waves, there might not be a vaccination that works. Currently, while awaiting the development of a vaccination that works, there is a

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significant, globally coordinated effort to create vaccines, mostly through the Coalition for Epidemic Preparedness Innovations. Vaccination has been an effective strategy in combating the spread of infectious diseases, e.g., pertussis, measles, and influenza [5]. To find out whether this disease will fade away after using these vaccines, deterministic mathematical models have been created that determine whether this disease will spread or fade away, because mathematical models can be easily understood and give specific results about the outbreak of the disease, however, as mentioned earlier, there is a need to introduce a random factor(s) to reflect the impact of fluctuating biological processes, which led to the emergence of stochastic models. After several studies, it was found that stochastic models give more accurate predictive results than deterministic mathematical models [6]. So in this work, a stochastic mathematical model for Covid-19 was formulated from the deterministic model, in order to know the correct extension of deterministic models to their stochastic counterparts, and to take advantage of the enormous amount of deterministic models through converting them stochastic models.

2- MATERIALS and METHODS

2.1 The deterministic model:

The model is divided into eight compartmental classes [7], namely, susceptible (S), first dose vaccinated (V_1), second dose vaccinated (V_2), exposed (E), asymptomatic (A), symptomatic (I), hospitalized (H), and recovery (R). The flowchart of the formulated model given in Figure 1, shows the transition between the compartments.

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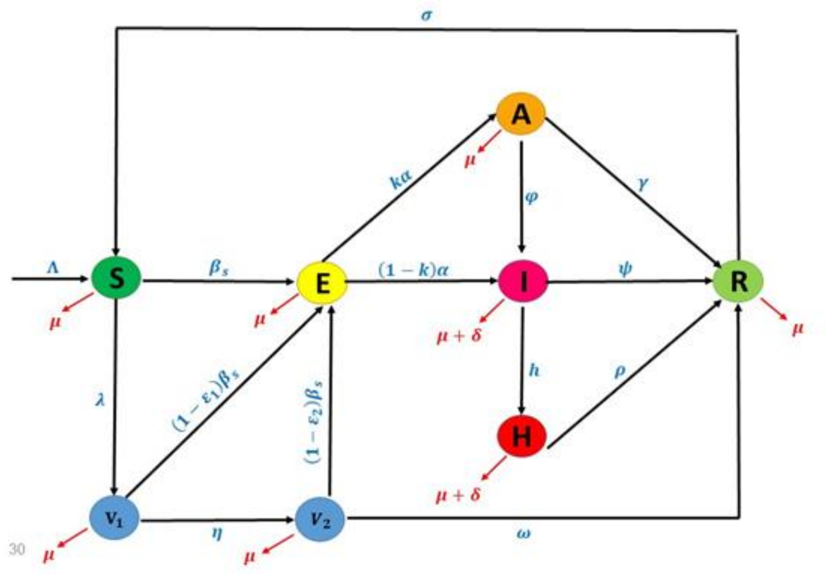

Figure 1. Flowchart of Covid 19 model

Therefore, the model is mathematically formulated as [8]

$$\begin{cases} \frac{dS}{dt} = \Lambda + \sigma R - [\beta_S + \lambda + \mu]S \\ \frac{dV_1}{dt} = \lambda S - [(1-\varepsilon_1)\beta_S + \eta + \mu]V_1 \\ \frac{dV_2}{dt} = \eta V_1 - [(1-\varepsilon_2)\beta_S + \omega + \mu]V_2 \\ \frac{dE}{dt} = \beta_S S + (1-\varepsilon_1)\beta_S V_1 + (1-\varepsilon_2)\beta_S V_2 - (\alpha+\mu)E \\ \frac{dA}{dt} = \kappa\alpha E - [\varphi + \gamma + \mu]A \\ \frac{dI}{dt} = (1-\kappa)\alpha E + \varphi A - [\psi + h + \mu + \delta]I \\ \frac{dH}{dt} = hI - [\rho + \mu + \delta] \\ \frac{dR}{dt} = \gamma A + \psi I + \rho H + \omega V_2 - (\sigma+\mu)R \end{cases} \quad (1)$$

Where, all the variables and parameters used in this model are described in Table (1).

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Due to the size of the model, the next generation matrix method (NGMM) will be used to calculate the basic reproduction number (R_0), this method can be summarized [9-11] as follows:

Table (1) Detailed description of state variables and relevantparameters of the proposed model

Variable	Description
$S(t)$	Susceptible
$V_1(t)$	First dose vaccinated
$V_2(t)$	Second dose vaccinated
$E(t)$	Exposed
$A(t)$	Asymptomatic
$I(t)$	Symptomatic
$H(t)$	Hospitalized
$R(t)$	Recovered
Λ	Hospitalized Recovered Recruitment rate into susceptible population
σ	Rate of loss of immunity
β_α	Rate of transmission from S to E due to contact with A
β_i	Rate of transmission from S to due E to contact with$I$
β_s	$= \beta_\alpha A + \beta_i I$
λ	Rate of vaccinated with first dose
μ	Natural human death rate
ε_1	Efficacy of the first dose
ε_2	Efficacy of the second dose
η	Rate of transmission from$V_1$to V_2
ω	Rate of transmission from V_2 to R
α	Progression rate from E to either A or$I$
κ	Proportion of asymptomatic infectious people
φ	Rate of transmission from A to I
γ	Rate of recovery of the asymptomatic human population
ψ	Rate of recovery of the symptomatic population
h	Rate of transmission from I to treatment
δ	Rate of death due to the COVID-19 disease
ρ	Rate of recovery due to treatment

1- Consider the infection subsystem, which contains only the infected compartmental classes that is:

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$$\frac{dE}{dt} = \beta_S S + (1-\varepsilon_1)\beta_S V_1 + (1-\varepsilon_2)\beta_S V_2 - (\alpha+\mu)E$$

$$\frac{dA}{dt} = \kappa\alpha E - [\varphi+\gamma+\mu]A$$

$$\frac{dI}{dt} = (1-\kappa)\alpha E + \varphi A - [\psi+h+\mu+\delta]I$$

$$\frac{dH}{dt} = hI - [\rho+\mu+\delta]$$

2- Decompose the Jacobian matrix of the infection subsystem into two matrices, $\mathcal{F}$ and $\mathcal{M}$ where $\mathcal{F}$ is the transmission matrix, and $\mathcal{M}$ is the transition matrix, that is $\mathcal{F}$ contains the entries corresponding to transmission events, where an epidemiological birth occurs, and $\mathcal{M}$ contains the entries corresponding to all other changes of state (including death)

$$\mathcal{F} = \begin{bmatrix} \beta_S S + (1-\varepsilon_1)\beta_S V_1 + (1-\varepsilon_2)\beta_S V_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\mathcal{M} = \begin{bmatrix} -(\alpha+\mu)E \\ \kappa\alpha E - [\varphi+\gamma+\mu]A \\ (1-\kappa)\alpha E + \varphi A - [\psi+h+\mu+\delta]I \\ hI - [\rho+\mu+\delta] \end{bmatrix}.$$

The disease free equilibrium is:

$$\text{DFE}= (S_0 V_{10} V_{20}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0);$$

$$S_0 = \frac{\Lambda}{\mu+\lambda} \tag{2}$$

$$V_{10} = \frac{\lambda}{\mu+\eta} S_0 = \frac{\lambda\Lambda}{(\mu+\eta)(\mu+\lambda)} \tag{3}$$

$$V_{20} = \frac{\eta}{\mu+\omega} V_{10} = \frac{\eta}{\mu+\omega} \cdot \frac{\lambda\Lambda}{(\mu+\eta)(\mu+\lambda)} \tag{4}$$

The Jacobian at decease free equilibrium

$F =$

$$\begin{bmatrix} 0 & \beta_A S_0 + (1-\varepsilon_1)\beta_A V_{10} + (1-\varepsilon_2)\beta_A V_{20} & \beta_I S_0 + (1-\varepsilon_1)\beta_I V_{10} + (1-\varepsilon_2)\beta_I V_{20} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} -(\alpha+\mu) & 0 & 0 & 0 \\ k\alpha & -[\varphi+\gamma+\mu] & 0 & 0 \\ (1-k)\alpha & \varphi & -[\psi+h+\mu+\delta] & 0 \\ 0 & 0 & h & -[\rho+\mu+\delta] \end{bmatrix}.$$

Since the matrix$F$ has three zero rows, the next generation matrix of system (1) is taken by the spectral radiusK_c of the NGM which in this case reduces to single element, as follows:

$$K_c = (-E^T F)(M^{-1}E)$$

Where: $E = (1 \quad 0 \quad 0 \quad 0)^T$

$$K_c = -(0 \quad \beta_A S_0 + (1-\varepsilon_1)\beta_A V_{10} + (1-\varepsilon_2)\beta_A V_{20} \beta_I S_0 + (1-\varepsilon_1)\beta_I V_{10} + (1-\varepsilon_2)\beta_I V_{20} \quad 0)$$

$$\begin{pmatrix} \frac{-1}{\alpha+\mu} \\ \frac{-k\alpha}{(\alpha+\mu)(\varphi+\gamma+\mu)} \\ \frac{-1}{\psi+h+\mu+\delta}\left[\frac{(1-k)\alpha}{\alpha+\mu} + \frac{k\alpha\varphi}{(\alpha+\mu)(\varphi+\gamma+\mu)}\right] \\ \left\{\frac{-1}{\psi+h+\mu+\delta}\left[\frac{(1-k)\alpha}{\alpha+\mu} + \frac{k\alpha\varphi}{(\alpha+\mu)(\varphi+\gamma+\mu)}\right]\right\}\left\{\frac{-h}{\rho+\mu+\delta}\right\} \end{pmatrix}.$$

Therefore, the basic reproduction number of the system is given by [12]:

$$R_0 = [S_0 + (1-\varepsilon_1)V_{10} + (1-\varepsilon_2)V_{20}]\left\{\beta_A\left(\frac{k\alpha}{(\alpha+\mu)(\varphi+\gamma+\mu)}\right) + \beta_I\left(\frac{\alpha[\varphi+(1-k)(\gamma+\mu)]}{(\psi+h+\mu+\delta)(\alpha+\mu)(\varphi+\gamma+\mu)}\right)\right\}.$$

3- Numerical Representation

To support the mathematical analysis of the proposed model, the numerical simulations are carried out using MATLAP tools (see the Appendix) to represent solutions of deterministic and stochastic models shown in figure 2. and figure 3. respectively.

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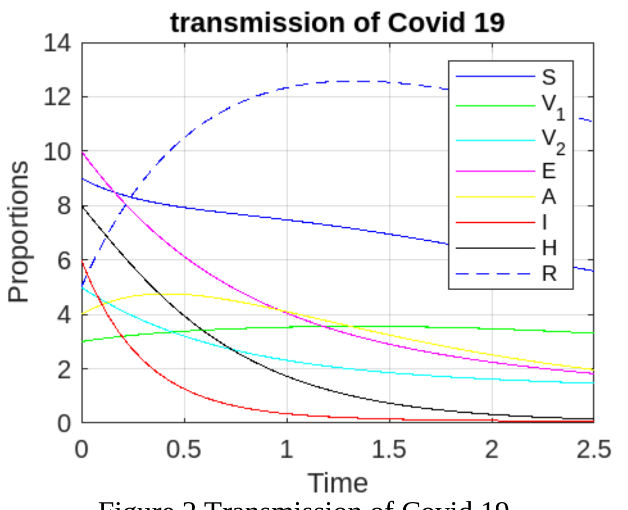

Figure 2.Transmission of Covid 19,
$\sigma = 0.2, \beta_S = 0.3, \lambda = 0.4, \mu = 0.3, \varepsilon_1 = 1, \eta = 0.5, \varepsilon_2 = 1,$
$\omega = 1, \alpha = 1, \kappa = 1, \varphi = 0.5, \gamma = 1, \psi = 1, h = 1, \delta = 1, \rho = 0.6.$

4- The stochastic model for system (1):
Rewrite the model as

$$\begin{cases} \frac{dS}{dt} = \sigma R - \beta_S S - \lambda S - \mu S \\ \frac{dV_1}{dt} = \lambda S - (1-\varepsilon_1)\beta_S V_1 - \eta V_1 - \mu V_1 \\ \frac{dV_2}{dt} = \eta V_1 - (1-\varepsilon_2)\beta_S V_2 - \omega V_2 - \mu V_2 \\ \frac{dE}{dt} = \beta_S S + (1-\varepsilon_1)\beta_S V_1 + (1-\varepsilon_2)\beta_S V_2 - \alpha E - \mu E \\ \frac{dA}{dt} = \kappa\alpha E - \varphi A - \gamma A - \mu A \\ \frac{dI}{dt} = \alpha E - \kappa\alpha E + \varphi A - \psi I - hI - \mu I - \delta I \\ \frac{dH}{dt} = hI \\ \frac{dR}{dt} = \gamma A + \psi I + \rho H + \omega V_2 - \sigma R - \mu R \end{cases} \quad (5)$$

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1- Probabilities associated with changes in the transmission of Covid 19 model are calculated in Table (2).

Table (2).Probabilities associated with changes in the transmissionof Covid 19 model

Changes,Δx_i	Probability,p_i
$(1,0,0,0,0,0,0,-1)^{tr}$.	$\sigma R\ \Delta t.$
$(-1,0,0,1,0,0,0,0)^{tr}$.	$\beta_S S\ \Delta t.$
$(-1,1,0,0,0,0,0,0)^{tr}$.	$\lambda S\ \Delta t.$
$(-1,0,0,0,0,0,0,0)^{tr}$.	□$S\ \Delta t.$
$(0,-1,0,1,0,0,0,0)^{tr}$.	$(1-\varepsilon_1)\beta_S V_1\ \Delta t.$
$(0,-1,1,0,0,0,0,0)^{tr}$.	$\eta V_1\ \Delta t.$
$(0,-1,0,0,0,0,0,0)^{tr}$.	□$V_1\ \Delta t.$
$(0,0,-1,1,0,0,0,0)^{tr}$.	$(1-\varepsilon_2)\beta_S V_2\ \Delta t.$
$(0,0,-1,0,0,0,0,1)^{tr}$.	$\omega V_2\ \Delta t.$
$(0,0,-1,0,0,0,0,0)^{tr}$.	□$V_2\ \Delta t.$
$(0,0,0,-1,0,1,0,0)^{tr}$.	$\alpha E\Delta t.$
$(0,0,0,-1,0,0,0,0)^{tr}$.	□$E\Delta t.$
$(0,0,0,0,1,-1,0,0)^{tr}$.	$\kappa\alpha E\ \ \Delta t.$
$(0,0,0,0,-1,1,0,0)^{tr}$.	$\varphi A\ \ \Delta t.$
$(0,0,0,0,-1,0,0,1)^{tr}$.	$\gamma A\ \ \Delta t.$
$(0,0,0,0,-1,0,0,0)^{tr}$.	□$A\Delta t.$
$(0,0,0,0,0,-1,0,1)^{tr}$.	$\psi I\ \ \Delta t.$
$(0,0,0,0,0,-1,1,0)^{tr}$.	$hI\ \ \Delta t.$
$(0,0,0,0,0,-1,0,0)^{tr}$.	$\mu I\ \ \Delta t.$
$(0,0,0,0,0,-1,0,0)^{tr}$.	$\delta I\ \ \Delta t.$
$(0,0,0,0,0,0,0,1)^{tr}$.	$\rho H\ \Delta t.$
$(0,0,0,0,0,0,0,-1)^{tr}$.	□$R\ \Delta t.$

2- The expectation $E(\Delta x)=\sum_{i=1}^{22} p_i \Delta x_i$ is 8×1 vector. To order Δt, the expectation can be expressed as follows:

$$E(\Delta x)=\sum_{i=1}^{22} p_i \Delta x_i = p_1\Delta x_1 + p_2\Delta x_2 + p_3\Delta x_3 + \cdots + p_{22}\Delta x_{22}.$$

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$$E(\Delta x)=\sum_{i=1}^{22}p_i\Delta x_i \;=\sigma R\begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\-1\end{pmatrix}+\beta_S S\begin{pmatrix}-1\\0\\0\\1\\0\\0\\0\\0\end{pmatrix}+\lambda S\begin{pmatrix}-1\\1\\0\\0\\0\\0\\0\\0\end{pmatrix}+$$

$$\mu S\begin{pmatrix}-1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}+(1-\varepsilon_1)\beta_S V_1\begin{pmatrix}0\\-1\\0\\1\\0\\0\\0\\0\end{pmatrix}+\eta V_1\begin{pmatrix}0\\-1\\1\\0\\0\\0\\0\\0\end{pmatrix}+\mu V_1\begin{pmatrix}0\\-1\\0\\0\\0\\0\\0\\0\end{pmatrix}+$$

$$(1-\varepsilon_2)\beta_S V_2\begin{pmatrix}0\\0\\-1\\1\\0\\0\\0\\0\end{pmatrix}+\omega V_2\begin{pmatrix}0\\0\\-1\\0\\0\\0\\0\\1\end{pmatrix}+V_2\begin{pmatrix}0\\0\\-1\\0\\0\\0\\0\\0\end{pmatrix}+\alpha E\begin{pmatrix}0\\0\\0\\-1\\0\\1\\0\\0\end{pmatrix}+$$

$$\mu E\begin{pmatrix}0\\0\\0\\-1\\0\\0\\0\\0\end{pmatrix}+\kappa\alpha E\begin{pmatrix}0\\0\\0\\0\\1\\-1\\0\\0\end{pmatrix}+\varphi A\begin{pmatrix}0\\0\\0\\0\\-1\\1\\0\\0\end{pmatrix}+\gamma A\begin{pmatrix}0\\0\\0\\0\\-1\\0\\0\\1\end{pmatrix}+$$

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$$\mu A\begin{pmatrix}0\\0\\0\\0\\-1\\0\\0\\0\end{pmatrix}+\psi I\begin{pmatrix}0\\0\\0\\0\\0\\-1\\0\\1\end{pmatrix}+hI\begin{pmatrix}0\\0\\0\\0\\0\\-1\\1\\0\end{pmatrix}+\mu I\begin{pmatrix}0\\0\\0\\0\\0\\-1\\0\\0\end{pmatrix}+\delta I\begin{pmatrix}0\\0\\0\\0\\0\\-1\\0\\0\end{pmatrix}+$$

$$\rho H\begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix}+\mu R\begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\-1\end{pmatrix}.$$

$$E(\Delta x)=\begin{pmatrix}\sigma R\\0\\0\\0\\0\\0\\0\\-\sigma R\end{pmatrix}+\begin{pmatrix}-\beta_S S\\0\\0\\\beta_S S\\0\\0\\0\\0\end{pmatrix}+\begin{pmatrix}-\lambda S\\\lambda S\\0\\0\\0\\0\\0\\0\end{pmatrix}+\begin{pmatrix}-\mu S\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}+$$

$$\begin{pmatrix}0\\-(1-\varepsilon_1)\beta_S V_1\\0\\(1-\varepsilon_1)\beta_S V_1\\0\\0\\0\\0\end{pmatrix}+\begin{pmatrix}0\\-\eta V_1\\\eta V_1\\0\\0\\0\\0\\0\end{pmatrix}+\begin{pmatrix}0\\-\mu V_1\\0\\0\\0\\0\\0\\0\end{pmatrix}+$$

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$$\begin{pmatrix} 0 \\ 0 \\ -(1-\varepsilon_2)\beta_S V_2 \\ (1-\varepsilon_2)\beta_S V_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega V_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ \omega V_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\mu V_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\alpha E \\ 0 \\ \alpha E \\ 0 \\ 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\mu E \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \kappa\alpha E \\ -\kappa\alpha E \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\varphi A \\ \varphi A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\gamma A \\ 0 \\ 0 \\ \gamma A \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\mu A \\ 0 \\ 0 \\ 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\psi I \\ 0 \\ \psi I \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -hI \\ hI \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\mu I \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\delta I \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \rho H \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\mu R \end{pmatrix}.$$

$$E(\Delta x) = \begin{pmatrix} \sigma R - \beta_S S - \lambda S - \mu S \\ \lambda S - (1-\varepsilon_1)\beta_S V_1 - \eta V_1 - \mu V_1 \\ \eta V_1 - (1-\varepsilon_2)\beta_S V_2 - \omega V_2 - \mu V_2 \\ \beta_S S + (1-\varepsilon_1)\beta_S V_1 + (1-\varepsilon_2)\beta_S V_2 - \alpha E - \mu E \\ \kappa\alpha E - \varphi A - \gamma A - \mu A \\ \alpha E - \kappa\alpha E + \varphi A - \psi I - hI - \mu I - \delta I \\ hI \\ \gamma A + \psi I + \rho H + \omega V_2 - \sigma R - \mu R \end{pmatrix} \Delta t.$$

3- The diffusion matrix G of dimension 8×22 is:

$G =$

$$
\left(\begin{array}{cccccccccccccccccccccc}
\sqrt{\sigma R} & -\sqrt{\beta_S S} & -\sqrt{\lambda S} & -\sqrt{\mu S} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{\lambda S} & 0 & -\sqrt{(1-\varepsilon_1)\beta_S V_1} & -\sqrt{\eta V_1} & -\sqrt{\mu V_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{\eta V_1} & 0 & -\sqrt{(1-\varepsilon_2)\beta_S V_2} & -\sqrt{\omega V_2} & -\sqrt{\mu V_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sqrt{\beta_S S} & 0 & 0 & \sqrt{(1-\varepsilon_1)\beta_S V_1} & 0 & 0 & \sqrt{(1-\varepsilon_2)\beta_S V_2} & 0 & 0 & -\sqrt{\alpha E} & -\sqrt{\mu E} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\kappa\alpha E} & -\sqrt{\varphi A} & -\sqrt{\gamma A} & -\sqrt{\mu A} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\alpha E} & 0 & -\sqrt{\kappa\alpha E} & \sqrt{\varphi A} & 0 & 0 & -\sqrt{\psi I} & -\sqrt{hI} & -\sqrt{\mu I} & -\sqrt{\delta I} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{hI} & 0 & 0 & 0 & 0 \\
-\sqrt{\sigma R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\omega V_2} & 0 & 0 & 0 & 0 & 0 & \sqrt{\gamma A} & 0 & \sqrt{\psi I} & 0 & 0 & 0 & \sqrt{\rho H} & -\sqrt{\mu R}
\end{array}\right).
$$

4- we formulate the stochastic system as:

$$dX(t) = f(X(t), t)dt + g(X(t), t)dW(t). \quad (6)$$

Where:

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$$dX(t)=\begin{bmatrix} dS_t \\ dV_{1_t} \\ dV_{2_t} \\ dE_t \\ dA_t \\ dI_t \\ dH_t \\ dR_t \end{bmatrix}, f(X(t),t)=\left[\frac{E(\Delta X)}{\Delta t}\right], g(X(t),t)$$

$$= G \text{ and } dW(t)=\begin{bmatrix} dW_1(t) \\ dW_2(t) \\ \cdot \\ \cdot \\ \cdot \\ dW_{22}(t) \end{bmatrix}$$

Thus, the system takes the following form:

$$\begin{pmatrix} dS_t \\ dV_{1_t} \\ dV_{2_t} \\ dE_t \\ dA_t \\ dI_t \\ dH_t \\ dR_t \end{pmatrix} = \begin{pmatrix} \sigma R-\beta_S S-\lambda S-\mu S \\ \lambda S-(1-\varepsilon_1)\beta_S V_1-\eta V_1-\mu V_1 \\ \eta V_1-(1-\varepsilon_2)\beta_S V_2-\omega V_2-\mu V_2 \\ \beta_S S+(1-\varepsilon_1)\beta_S V_1+(1-\varepsilon_2)\beta_S V_2-\alpha E-\mu E \\ \kappa\alpha E-\varphi A-\gamma A-\mu A \\ \alpha E-\kappa\alpha E+\varphi A-\psi I-hI-\mu I-\delta I \\ hI \\ \gamma A+\psi I+\rho H+\omega V_2-\sigma R-\mu R \end{pmatrix} dt \; +$$

$$G = \begin{pmatrix} \sqrt{\sigma R} & -\sqrt{\beta_S S} & -\sqrt{\lambda S} & -\sqrt{\mu S} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\lambda S} & 0 & -\sqrt{(1-\varepsilon_1)\beta_S V_1} & -\sqrt{\eta V_1} & -\sqrt{\mu V_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\eta V_1} & 0 & -\sqrt{(1-\varepsilon_2)\beta_S V_2} \\ 0 & \sqrt{\beta_S S} & 0 & 0 & \sqrt{(1-\varepsilon_1)\beta_S V_1} & 0 & 0 & \sqrt{(1-\varepsilon_2)\beta_S V_2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\sigma R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$
\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\sqrt{\omega V_2} & -\sqrt{\mu V_2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\sqrt{\alpha E} & -\sqrt{\mu E} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sqrt{\kappa\alpha E} & -\sqrt{\varphi A} & -\sqrt{\gamma A} & -\sqrt{\mu A} \\
0 & 0 & \sqrt{\alpha E} & 0 & -\sqrt{\kappa\alpha E} & \sqrt{\varphi A} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{\omega V_2} & 0 & 0 & 0 & 0 & 0 & \sqrt{\gamma A} & 0
\end{matrix}
$$

$$
\left.\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-\sqrt{\psi I} & -\sqrt{hI} & -\sqrt{\mu I} & -\sqrt{\delta I} & 0 & 0 \\
0 & \sqrt{hI} & 0 & 0 & 0 & 0 \\
\sqrt{\psi I} & 0 & 0 & 0 & \sqrt{\rho H} & -\sqrt{\mu R}
\end{matrix}\right) \cdot \begin{pmatrix} dW_1(t) \\ dW_2(t) \\ . \\ . \\ . \\ dW_{22}(t) \end{pmatrix}.
$$

$$
\begin{aligned}
dS_t &= (\sigma R - \beta_S S - \lambda S - \mu S)dt + \\
&\sqrt{\sigma R}dW_1(t) - \sqrt{\beta_S S}dW_2(t) - \sqrt{\lambda S}dW_3(t) - \sqrt{\mu S}dW_4(t) \\
dV_{1_t} &= (\lambda S - (1-\varepsilon_1)\beta_S V_1 - \eta V_1 - \mu V_1) + \\
&\sqrt{\lambda S}dW_3(t) - \sqrt{(1-\varepsilon_1)\beta_S V_1}dW_5(t) - \sqrt{\eta V_1}dW_6(t) - \sqrt{\mu V_1}dW_7(t) \\
dV_{2_t} &= (\eta V_1 - (1-\varepsilon_2)\beta_S V_2 - \omega V_2 - \mu V_2)dt + \\
&\sqrt{\eta V_1}dW_6(t) - \sqrt{(1-\varepsilon_2)\beta_S V_2}dW_8(t) - \sqrt{\omega V_2}dW_9(t) - \sqrt{\mu V_2}dW_{10}(t) \\
dE_t &= (\beta_S S + (1-\varepsilon_1)\beta_S V_1 + (1-\varepsilon_2)\beta_S V_2 - \alpha E - \mu E)dt + \\
&\sqrt{\beta_S S}dW_2 + \sqrt{(1-\varepsilon_1)\beta_S V_1}dW_5 + \sqrt{(1-\varepsilon_2)\beta_S V_2}dW_8 - \sqrt{\alpha E}dW_{11} - \sqrt{\mu E}\, dW_{12} \\
dA_t &= (\kappa\alpha E - \varphi A - \gamma A - \mu A)dt + \\
&\sqrt{\kappa\alpha E}dW_{13}(t) - \sqrt{\varphi A}dW_{14}(t) - \sqrt{\gamma A}dW_{15}(t) - \sqrt{\mu A}dW_{16}(t) \qquad (7) \\
dI_t &= (\alpha E - \kappa\alpha E + \varphi A - \psi I - hI - \mu I - \delta I)dt + \sqrt{\alpha E}dW_{11} \\
&-\sqrt{\kappa\alpha E}dW_{13} + \sqrt{\varphi A}dW_{14} - \sqrt{\psi I}dW_{17} - \sqrt{hI}dW_{18} - \sqrt{\mu I}dW_{19}\sqrt{\delta I}\, dW_{20} \\
dH_t &= (hI\,)dt + \sqrt{hI}dW_{18}(t) \\
dR_t &= (\gamma A + \psi I + \rho H + \omega V_2 - \sigma R - \mu R\ )dt + \\
&\sqrt{\gamma A}dW_{15}(t) + \sqrt{\psi I}dW_{17}(t) + \sqrt{\rho H}dW_{21}(t) + \sqrt{\omega V_2}dW_9(t) - \sqrt{\sigma R}dW_1(t) - \sqrt{\mu R}\, dW_{22}(t)
\end{aligned}
$$

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5- Numerical Representation

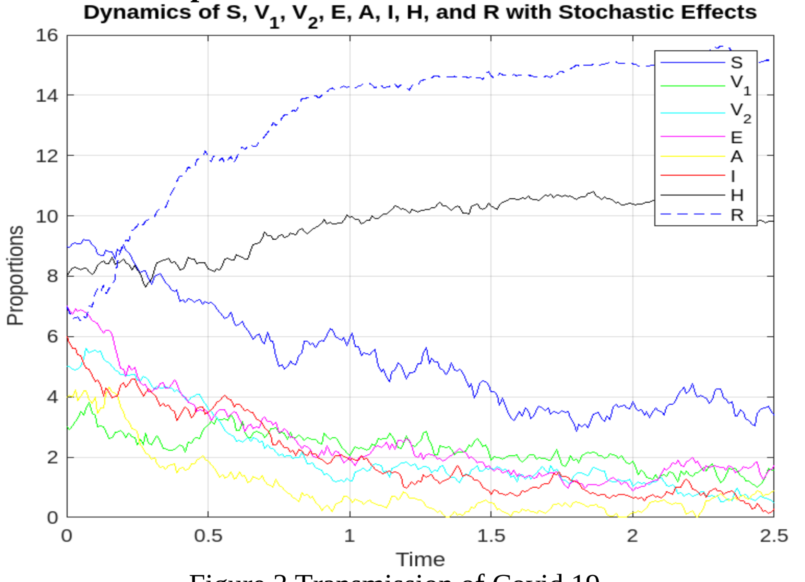

Figure 3.Transmission of Covid 19,

$$\sigma = 0.2, \beta_S = 0.3, \lambda = 0.4, \mu = 0.3, \varepsilon_1 = 1, \eta = 0.5, \varepsilon_2 = 0.8, \omega = 1,$$
$$\alpha = 1, \kappa = 0.5, \varphi = 0.5, \gamma = 1, \psi = 0.5, h = 0.5, \delta = 1, \rho = 0.6.$$

The generated stochastic model helps to understand how infectious disease spreads and show that spread can be unpredictable and highly dependent on individual interactions, contributing to informed decisions about public health measures.Rapid intervention activities, social distancing and vaccinations can have a significant impact in reducing the spread of the disease. By including stochastic terms, the accuracy of deterministic model can be improved to be more realistic in representing the complex dynamics of infection.

6- Conclusion:

The purpose of this research is to form stochastic mathematical models suitable for the study of epidemiology, to clarify the effect of adding random termsto deterministic models. It was concluded that stochastic models give accurate results and good prediction about the spread of diseases, as including random variables in deterministic biological models helps us to produce improve

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judgment in fields such as biological sciences and medicine by enhancing our knowledge of biological processes and their interrelationships. Interest in mathematical models of epidemiology has increased recently because of its health and economic implications.

It has been shown that many useful properties of solutions for epidemiology can be deduced using stochastic mathematical models. In this work, a stochastic system was studied on some vital mathematical models specialized in infectious diseases, which is Covid-19. Learn how to create stochastic model for its counterpart deterministic mode, and then solve these systems numerically and also highlighted using MATLAB tools. It is found that stochastic system gives greater accuracy and accurate prediction about the spread of the disease.

Recommendations for Farther Studies

We recommend taking advantage of literature stochastic calculus, in an attempt to study the stochastic model analytically, in future studies, to clarify more characteristics of its solution to know the conditions that guarantee disappearance of the epidemic.

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Appendix

6-1. Code of the transmission of Covid 19 (Deterministic model):

```
function simulate_SEIRAHVR
    % Parameters
    beta_S = 0.3; lambda =0.4; mu = 0.3;
    eta = 0.5; omega = 1; alpha = 1;
    kappa = 1; phi = 0.1; gamma = 1;
    psi = 1; h = 1; delta = 1;
    rho = 0.6; sigma = 0.2; epsilon1 = 1;
    epsilon2 = 1;

    % Initial conditions
    S0 = 9; V1_0 = 3; V2_0 = 5;
    E0 = 10; A0 = 4; I0 = 6;
    H0 = 8; R0 = 5;
    initial_conditions = [S0, V1_0, V2_0, E0, A0, I0, H0, R0];

    % Time span
    tspan = [0 2.5];

    % Solve ODEs
    [t, y] = ode45(@(t, y) odes(t, y, beta_S, lambda, mu, eta, omega, alpha,
kappa, phi, gamma, psi, h, delta, rho, sigma, epsilon1, epsilon2), tspan,
initial_conditions);

    % Extract results
    S = y(:, 1);
    V1 = y(:, 2);
    V2 = y(:, 3);
    E = y(:, 4);
    A = y(:, 5);
    I = y(:, 6);
    H = y(:, 7);
    R = y(:, 8);
```

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```
    % Plot results
    figure;
    plot(t, S, '-b', 'DisplayName', 'S');
    hold on;
    plot(t, V1, '-g', 'DisplayName', 'V_1');
    plot(t, V2, '-c', 'DisplayName', 'V_2');
    plot(t, E, '-m', 'DisplayName', 'E');
    plot(t, A, '-y', 'DisplayName', 'A');
    plot(t, I, '-r', 'DisplayName', 'I');
    plot(t, H, '-k', 'DisplayName', 'H');
    plot(t, R, '--b', 'DisplayName', 'R');
    xlabel('Time');
    ylabel('Proportions');
    title(' transmission of Covid 19 ');
    legend;
    grid on;
    hold off;

    function dydt = odes(t, y, beta_S, lambda, mu, eta, omega, alpha, kappa,
phi, gamma, psi, h, delta, rho, sigma, epsilon1, epsilon2)
        % State variables
        S = y(1);
        V1 = y(2);
        V2 = y(3);
        E = y(4);
        A = y(5);
        I = y(6);
        H = y(7);
        R = y(8);

        % Differential equations
        dS_dt = A + sigma * R - (beta_S + lambda + mu) * S;
        dV1_dt = lambda * S - ((1 - epsilon1) * beta_S + eta + mu) * V1;
        dV2_dt = eta * V1 - ((1 - epsilon2) * beta_S + omega + mu) * V2;
        dE_dt = beta_S * S + (1 - epsilon1) * beta_S * V1 + (1 - epsilon2) *
beta_S * V2 - (alpha + mu) * E;
        dA_dt = kappa * alpha * E - (phi + gamma + mu) * A;
        dI_dt = (1 - kappa) * alpha * E + phi * A - (psi + h + mu + delta) *
I;
        dH_dt = h * I - (rho + mu + delta) * H;
        dR_dt = gamma * A + psi * I + rho * H + omega * V2 - (sigma + mu) *
R;

        % Return derivatives
        dydt = [dS_dt; dV1_dt; dV2_dt; dE_dt; dA_dt; dI_dt; dH_dt; dR_dt];
    end
end
```

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2. Code of the transmission of Covid 19 (Stochastic model):

```
function simulate_SEIRAHVR_SDE
% Parameters
beta_S = 0.3; lambda = 0.4; mu = 0.3;
    eta = 0.5; omega = 1; alpha = 1;
    kappa = 0.5; phi = 0.5; gamma = 1;
    psi = 0.5; h = 0.5; delta = 1;
    rho = 0.6; sigma = 0.2; epsilon1 = 0.8;
    epsilon2 = 1;

% Initial conditions
    S0 = 9; V1_0 = 3; V2_0 = 5;
    E0 = 7; A0 = 4; I0 = 6;
    H0 = 8; R0 = 7;
    y0 = [S0, V1_0, V2_0, E0, A0, I0, H0, R0];

% Time span
tspan = [0 2.5];
    dt = 0.01;
    t = tspan(1):dt:tspan(2);

% Number of time steps
num_steps = length(t);

% Preallocate arrays for results
    y = zeros(num_steps, length(y0));
y(1, :) = y0;

% Simulate SDEs using Euler-Maruyama method
for i = 1:num_steps-1
drift_term = sde_drift(y(i, :), beta_S, lambda, mu, eta,
omega, alpha, kappa, phi, gamma, psi, h, delta, rho,
sigma, epsilon1, epsilon2);
diffusion_term = sde_diffusion(y(i, :), beta_S, lambda,
mu, eta, omega, alpha, kappa, phi, gamma, psi, h, delta,
rho, sigma, epsilon1, epsilon2) * (sqrt(dt) * randn(8,
1));

y_next = y(i, :) + drift_term * dt + diffusion_term';

% Ensure no negative values and discard imaginary parts
y_next = max(real(y_next), 0);
```

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```
y(i+1, :) = y_next;
end

% Extract results
    S = y(:, 1);
    V1 = y(:, 2);
    V2 = y(:, 3);
    E = y(:, 4);
    A = y(:, 5);
    I = y(:, 6);
    H = y(:, 7);
    R = y(:, 8);

% Plot results
    figure;
plot(t, S, '-b', 'DisplayName', 'S');
    hold on;
plot(t, V1, '-g', 'DisplayName', 'V_1');
plot(t, V2, '-c', 'DisplayName', 'V_2');
plot(t, E, '-m', 'DisplayName', 'E');
plot(t, A, '-y', 'DisplayName', 'A');
plot(t, I, '-r', 'DisplayName', 'I');
plot(t, H, '-k', 'DisplayName', 'H');
plot(t, R, '--b', 'DisplayName', 'R');
xlabel('Time');
ylabel('Proportions');
title('Dynamics of S, V_1, V_2, E, A, I, H, and R with
Stochastic Effects');
    legend;
    grid on;
    hold off;

function drift = sde_drift(y, beta_S, lambda, mu, eta,
omega, alpha, kappa, phi, gamma, psi, h, delta, rho,
sigma, epsilon1, epsilon2)
% State variables
        S = y(1);
        V1 = y(2);
        V2 = y(3);
        E = y(4);
        A = y(5);
        I = y(6);
        H = y(7);
```

```
        R = y(8);
% Drift terms (deterministic part)
dS_dt = sigma * R - beta_S * S - lambda * S - mu * S;
        dV1_dt = lambda * S - (1 - epsilon1) * beta_S * V1
- eta * V1 - mu * V1;
        dV2_dt = eta * V1 - (1 - epsilon2) * beta_S * V2 -
omega * V2 - mu * V2;
dE_dt = beta_S * S + (1 - epsilon1) * beta_S * V1 + (1 -
epsilon2) * beta_S * V2 - alpha * E - mu * E;
dA_dt = kappa * alpha * E - phi * A - gamma * A - mu * A;
dI_dt = (1 - kappa) * alpha * E + phi * A - psi * I - h *
I - mu * I - delta * I;
dH_dt = h * I;
dR_dt = gamma * A + psi * I + rho * H + omega * V2 - sigma
* R - mu * R;

        drift = [dS_dt; dV1_dt; dV2_dt; dE_dt; dA_dt;
dI_dt; dH_dt; dR_dt]';
end
function diffusion = sde_diffusion(y, beta_S, lambda, mu,
eta, omega, alpha, kappa, phi, gamma, psi, h, delta, rho,
sigma, epsilon1, epsilon2)
% State variables
        S = y(1);
        V1 = y(2);
        V2 = y(3);
        E = y(4);
        A = y(5);
        I = y(6);
        H = y(7);
        R = y(8);
% Diffusion terms (stochastic part)
        diffusion = zeros(8, 8);
diffusion(1, 1) = sqrt(max(sigma * R, 0));
diffusion(2, 2) = sqrt(max(lambda * S, 0));
diffusion(3, 3) = sqrt(max(eta * V1, 0));
diffusion(4, 4) = sqrt(max(beta_S * S, 0));
diffusion(5, 5) = sqrt(max(kappa * alpha * E, 0));
diffusion(6, 6) = sqrt(max((1 - kappa) * alpha * E, 0));
diffusion(7, 7) = sqrt(max(h * I, 0));
diffusion(8, 8) = sqrt(max(gamma * A, 0));
end
```